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SEMI-EMPIRICAL METHOD OF ANALYSIS OF DIFFUSE RUNOFF OF CONSERVATIVE POLLUTANTS IN SMALL RIVERS: 2. INFLUENCE OF THE NUMBER OF MEASUREMENTS ON THE RESULTS OF THE INVERSE PROBLEM SOLUTION

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ABSTRACT

Relevance. When solving inverse problems of reconstruction spatial distribution of diffuse pollutant flow characteristics in small rivers, there arises inevitably the question related to the required number of design points. For the correct mathematical analysis, the number of design points used should be very significant (hundreds). A real field experiment, as a rule, is limited to a small number of points (dozens). However, at present, the influence of the number of design points on the stability of solving inverse problems for the equation of transporting conservative pollutants is poorly studied, although this issue is a key one when comparing the data of mathematical calculation and field measurements. In the presented paper we analyze the influence of the number of measurements of dissolved pollutant concentration on the value of its total mass flow rate in the process of diffuse runoff in small rivers. **Methods.** Calculation was based on the Monte Carlo method with randomization of pollutant concentration values. **Results.** When carrying out numerical experiments for a given set of test functions it was found that stable values of the required parameters (the total mass flow rate and the Peclet number) depend on the number of design points determining the spatial distribution of pollutant concentration on the site under study. It was found out that the stable values of parameters are determined by the number of points, five to ten times exceeding the number of unknown coefficients of the approximating polynomial. Dependence of values of required parameters on the number of design points for the results of field measurements was analyzed.

KEYWORDS: diffuse runoff, pollutant concentration, total mass flow rate, Peclet number, number of measurements

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INTRODUCTION

As follows from numerous studies devoted to solving inverse problems of reconstruction diffuse runoff of pollutants in polluted small rivers [1-9], the question of the number of required measurements of pollutant concentration distribution is poorly studied. When carrying out a numerical experiment, with preset random deviations, there are issues related to incorrectness of the problem under study (these problems are discussed, for example, in research [10]). As a result, in this case it is necessary to use such and such methods of regularization, which require a large number of data (as a rule, hundreds of design points [11]).

In the process of mathematical modeling, researchers may not limit themselves in this respect, examining any number of points with the given value of pollutant concentration (usually this issue is not discussed in studies). For example, the regularization method used in [11] requires about 400 points to restore the total mass flow rate of the pollutant. As a rule, the number of points is determined by the integration step of the corresponding advection-diffusion equation. However, the amount of data in a real field experiment is limited [12-19]. There arises a reasonable question of how correct it is to apply inverse problem methods to a small number of field measurements. In research [20], the optimization method is used to evaluate the specific mass flow rate of pollutants and the corresponding Peclet numbers for six measurement points. It is shown that the analysis done allows us to obtain some effective parameter values. At the same time, studies devoted to direct comparison of calculation data and field measurement data in literature available have not been found. The analysis of this problem seems to be extremely important because its solution will clarify the above dilemma and make it possible to adapt the general calculation methodology to field measurement data.

Paper [10] considers methodology for calculating the total mass flow rate of a conservative pollutant getting to a watercourse at a given number of points determining concentration distribution (50 points). It is obvious that depending on the nature of the test function behavior (mass flow rate of the pollutant per unit length) stability of calculation results will depend on the number of observation points for the field or numerical experiment. The purpose of this study is to determine the limits of stable calculation results depending on the number of measurements. The calculation results given in [10] are used as initial data.

CALCULATION METHODS

In research [10], the following test functions for the specific mass flow rate of a pollutant in the interval of 0 to 1 were introduced:

$$\tilde{\varphi}(z) = 0{,}2 + 0{,}3z - 0{,}25z^2, \quad (1)$$

$$\tilde{\varphi}(z) = 0{,}2e^z + 0{,}2z^4, \quad (2)$$

$$\tilde{\varphi}(z) = 0{,}2 + 0{,}2e^{-\left(\frac{z-0{,}3}{0{,}1}\right)^2} + 0{,}45e^{-\left(\frac{z-0{,}7}{0{,}3}\right)^2}. \quad (3)$$

The inverse problem for reconstruction the total mass flow rate distribution was considered:

$$\tilde{\Phi}(z) = \int_0^z \tilde{\varphi}(z)dz. \quad (4)$$

In this case, function $\tilde{\Phi}(z)$ is represented as a polynomial:

$$\tilde{\Phi}(z) = \sum_{n=1}^{m} a_n z^n. \quad (5)$$

On the basis of the stationary advection-diffusion equation with the help of the nonlinear global optimization and Monte Carlo methods, it was found that for functions (1) – (3) the third degree of the polynomial gives quite stable and correct results of solving the inverse problem. For the algorithm of the Monte Carlo method there were used N = 1000 variants of random deviations of dimensionless concentration values in interval $\delta = \pm 0.05$. To analyze the influence of the number of measurements on the results of solving the inverse problem, the following methodology is applied in the present work. For each of functions (1) – (3) with the help of the Monte Carlo method at N = 1000, $\delta = \pm 0.05$ calculation was done in order to analyze stability of calculation results of the main parameters: the Peclet number α and the total mass flow rate of the pollutant over the whole interval under study – $\tilde{\Phi}(1)$. The number of calculation points – k for different degrees of the polynomial (5) is a variable parameter.

Fig. 1 shows calculation results of pollutant concentration distribution and the total mass flow rate for function (3) at different degrees of the polynomial and different numbers of calculation points at known values α = 10, θ_1 = 1,5, α being the Peclet number, θ_1 being the dimensionless pollutant concentration in the lower section [10].

At the given degree of the polynomial there was found a minimum number of points at which calculation still gives stable results (when using a smaller number of points, physically unrealizable values of the required parameters are observed, which under specified conditions of randomization begin to fluctuate in wide intervals). For m = 1 the minimum required number of points is 5, m = 2 – 10, m = 3 – 35. Fig. 1 shows that in domains of parametric stability, the result of the total mass flow restoration depends weakly on the number of design points. The largest visible deviations (about 15 per cent) are observed at k = 5 for the first degree of the polynomial (Fig. 1*a*). However, at k =10 deviations are less than 1 per cent. At m = 2, m = 3 curves practically coincide with each other and with the exact solution to the direct problem [10].

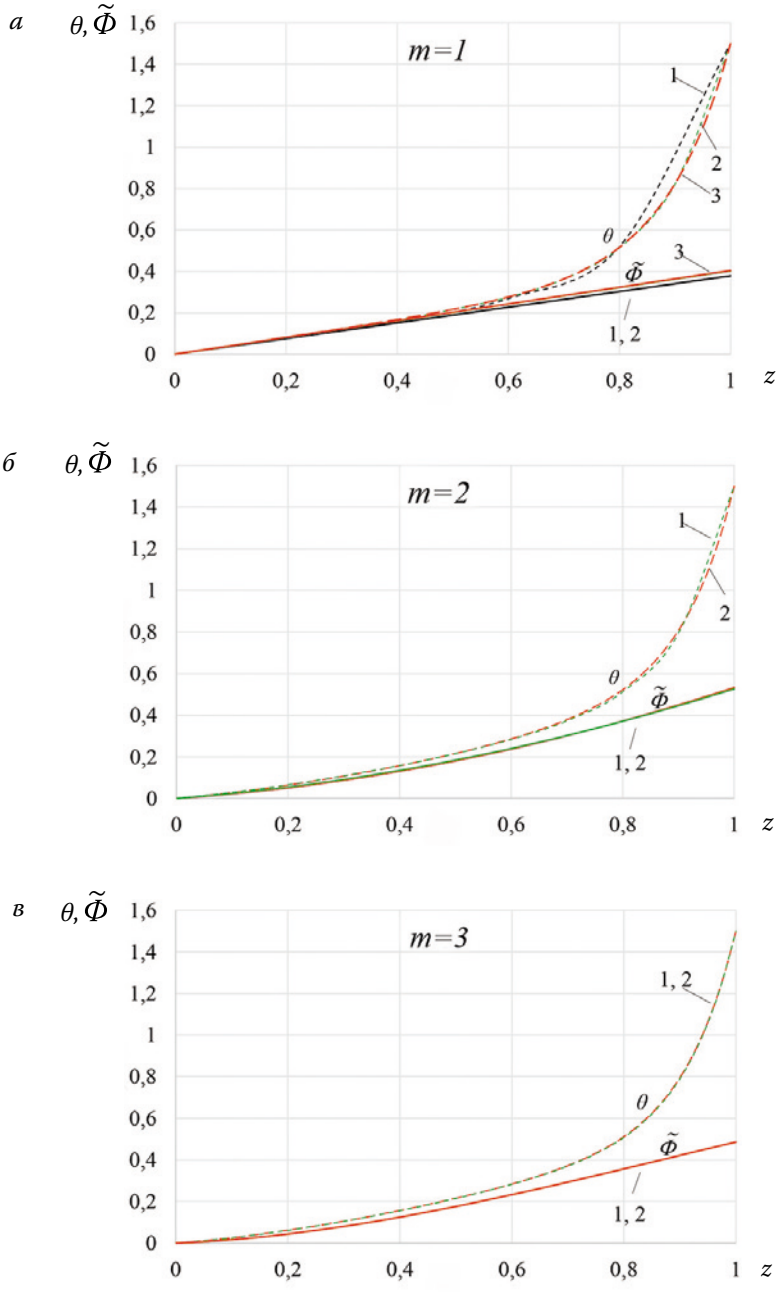


Fig. 1. Distribution of pollutant concentration θ and total mass flow rate $\tilde{\Phi}$ for function (3) at different degrees of the polynomial and different numbers of calculation points k: *a* – first degree of polynomial m=1, 1–k=5 , 2–k=10, 3–k=50; *b* – second-degree of polynomial m=2, 1–k=10, 2–k=50; *c* – third degree of polynomial m=3, 1–k=10, 2–k=50.

In [10], it was found that at the given randomization parameters for function (3), the value of the total flow rate is $\tilde{\Phi}(1) = 0.49$. For m=1, it is $\tilde{\Phi}(1) = 0.4$. However, for $m = 2$, $m = 3$ this value is close to the calculation value ($\tilde{\Phi}(1) \approx 0.5$). In [10], the exact problem solution gives $\tilde{\Phi}(1) \approx 0.46$. Thus, the second degree of the polynomial is quite sufficient for obtaining correct results when calculating for 10 or more points of the interval under study. It should be noted that calculation for test functions (1) and (2) gives similar results.

However, determination of the Peclet number and calculation of the total mass flow rate of the pollutant require a more detailed quantitative analysis. Fig. 2 shows the results of calculation of dependences of parameters α, $\tilde{\Phi}(1)$ on the number of calculation points for test function (1). It should be reminded that this function varies weakly with coordinate and distribution of specific mass flow rate in the interval under study is close to the homogeneous one.

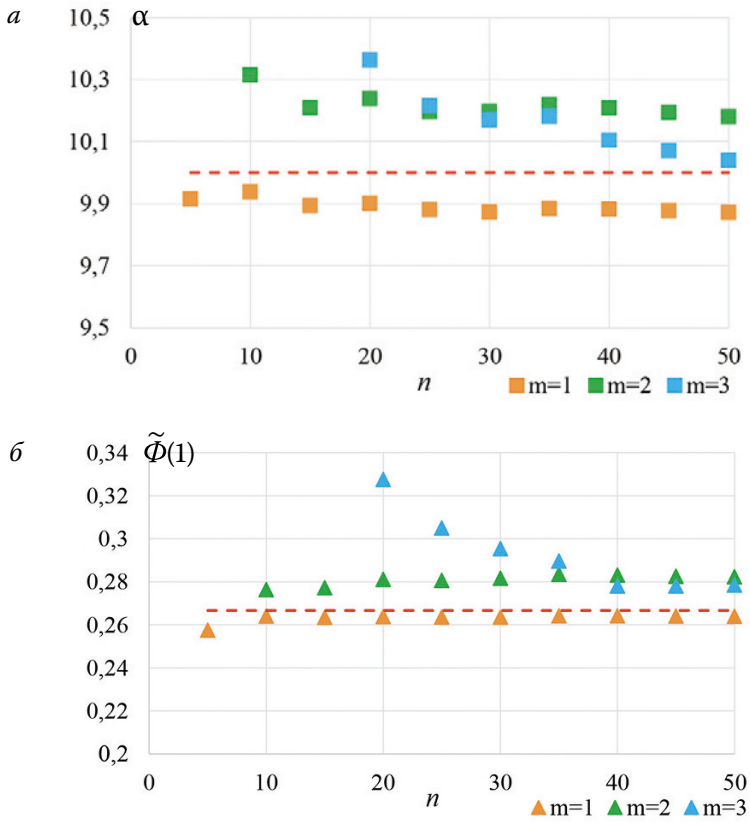


Fig. 2. Dependences of parameters α, $\tilde{\Phi}(1)$ on the number of calculation points for function (1) for various degrees of polynomials: *a* is reconstructed value of the Peclet number; *b* is reconstructed value of the total mass flow rate. Horizontal dotted lines determine the exact solution of the direct problem [10].

As follows from Fig. 2*a*, for the polynomial of the first degree, the value of the Peclet criterion is close to the exact value and weakly depends on the number of design points up to $k = 5$, calculation results being stable. The second degree of the polynomial gives comparatively overestimated values of α, calculation results being also stable towards changing the number of design points up to $k = 10$. The third degree of the polynomial gives the closest to the exact parameter value α at $k = 50$, but with a decrease in the number of design points, there is an increase in the value of the parameter, at $k = 20$ there being a loss of stability. It can be concluded that polynomials of the first and the second degrees give values of the mass flow rate that depend slightly on the number of points (Fig. 2*b*). Thus, despite the fact that polynomials of the first and the second degrees show quite correct and stable results, the most accurate parameter values are determined by the third degree of the polynomia at the number of points $k = 50$.

A different situation is observed for function (2) (Fig. 3): the corresponding dependence has an accelerated exponential growth. Calculation results are sta-

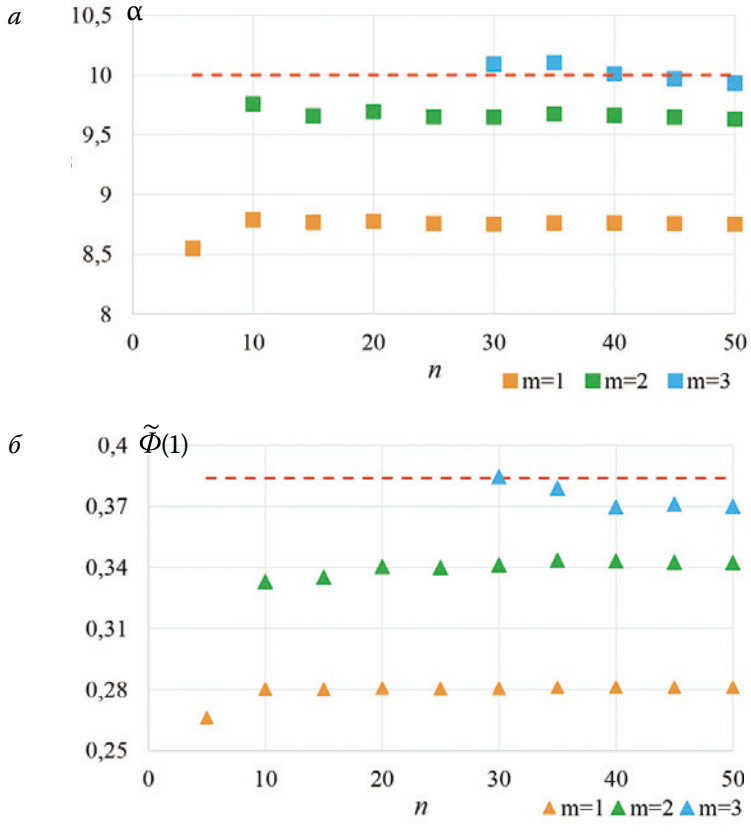


Fig. 3. Dependences of parameters α, $\tilde{\Phi}(1)$ on the number of calculation points for function (2) for various degrees of polynomials: *a* is the reconstructed value of the Peclet number; *b* is the reconstructed value of the total mass flow rate.

ble with respect to the number of points used, parameter α being most accurately determined with the third degree of the polynomial (Fig. 3*a*). In contrast to function (1), the first degree polynomial gives the worst result, while the second degree of the polynomialgives a satisfactory result, stability of the calculation determined with a minimum of ten points. The latter is explained by the fact that function (1) changes insignificantly in the interval under study, while function (2) is characterized by rapid growth making linear approximation unsuitable for it. Calculation of value $\tilde{\Phi}(1)$ gives similar results (Fig. 3*b*). For the third degree of the polynomial, stable results start from thirty points.

The results of calculating parameters α, $\tilde{\Phi}(1)$, for function (3) are shown in Fig. 4. The test function is nonmonotonic and has two maxima in the interval under study [10]. The second and third degree of the polynomials give close results both with respect to calculating the Peclet number and the total mass flow rate (Fig. 4), calculation results being also stable with respect to the number of calculation points. However, even in this case, the third degree of the polynomial is

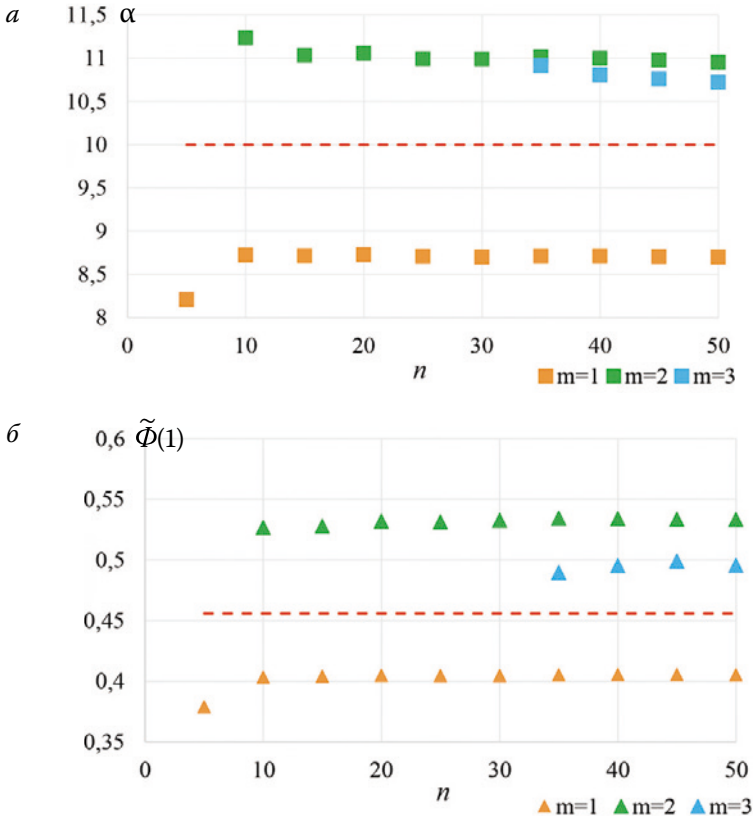


Fig. 4. Dependences of parameters α, $\tilde{\Phi}(1)$ on the number of calculation points for function (3) for various degrees of polynomials: *a* is the reconstructed value of the Peclet number; *b* is the reconstructed value of the total mass flow rate.

preferable at $k \geq 35$. Calculation shows that in all the cases considered, increasing the number of design points does not lead to changing parameter values for all degrees of polynomials. Here it should be noted that randomization of initial data does not enable us to get exact results determined when solving the direct problem, deviation of definable values from the exact ones being 5–7 per cent.

It is evident that the number of points necessary for correct calculation of the values examined above depends on the behavior of test functions. In all these cases, the most accurate results correspond to the third degree of the polynomial, which is the highest of degrees under study. However, for all functions except the simplest one (1), a relatively large number of points are necessary to obtain stable results. Indeed, for functions (2) and (3) stable results start at $k = 30$ and $k = 35$, respectively. The limiting factor is the highest degree of the polynomial at which randomization for the data available is possible in the interval under study. The higher the degree of the polynomial is the more accurate are the estimates of definable values. At the same time the degree of polynomial at which stable results are realized depends on the number of design points. It can be assumed that the criterion of correctness of calculation is the closeness of values of calculation parameters for quadratic and cubic equations and independence of their values from the number of design points.

In conclusion of this section, it should be noted that calculation for other values of the Peclet number and pollutant concentration at the right boundary θ_1 for the given test functions (1) – (3) leads to practically the same results with respect to stability of definable values.

Field Measurements

To apply the above methodology to the results of field measurements, the data of research [19] discussed in [10] were used. There were taken six measurements of dissolved cobalt (*Co*) concentration on the river section of 80 km length. The global minimum corresponding to the first degree of polynomial (5) at $a_1 = 0.228$, $\alpha = 7.3$ was determined using nonlinear optimization method. To verify the correctness of the calculated values, it was proposed to increase the number of design points up to $k = 100$ applying further the Monte Carlo method with randomization parameters ($N = 1000$, $\delta = \pm 0.05$). Fig. 5 shows distribution of concentration and total mass flow rate of pollutant for the number of points under the given randomization conditions.

As it follows from Fig. 5, results of the concentration distribution calculation depend weakly on the degree of the polynomial, which is characteristic of the incorrect inverse problem. However, when increasing the degree of the polynomial, for the total mass flow rate distribution convergence of simulations to a certain limiting dependence defined by value $\widetilde{\Phi}(1) \approx 0.3$ is clearly observed. The latter is illustrated in more detail in Fig. 6, which shows the corresponding dependences of parameters on the number of design points.

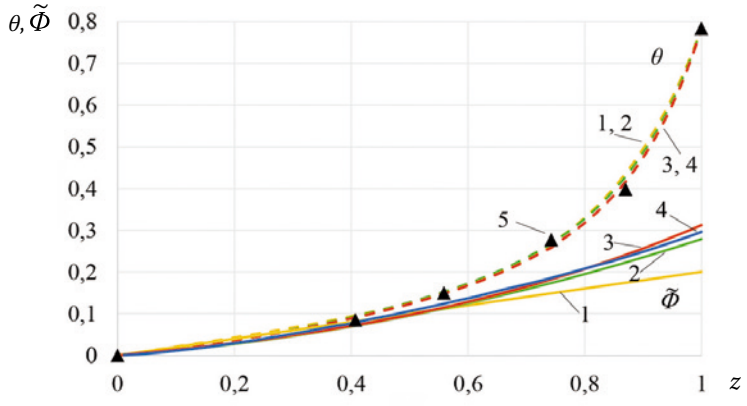


Fig. 5. Distribution of dissolved Co concentration **θ** and corresponding total mass flow rate $\tilde{\Phi}$ in the Hawkesbury–Nepean River section [19], obtained using the nonlinear optimization method at k=100 with different degrees of the polynomial (5): 1–m=1, 2–m=2, 3–m=3, 4–m=4; 5 is results of field experiment.

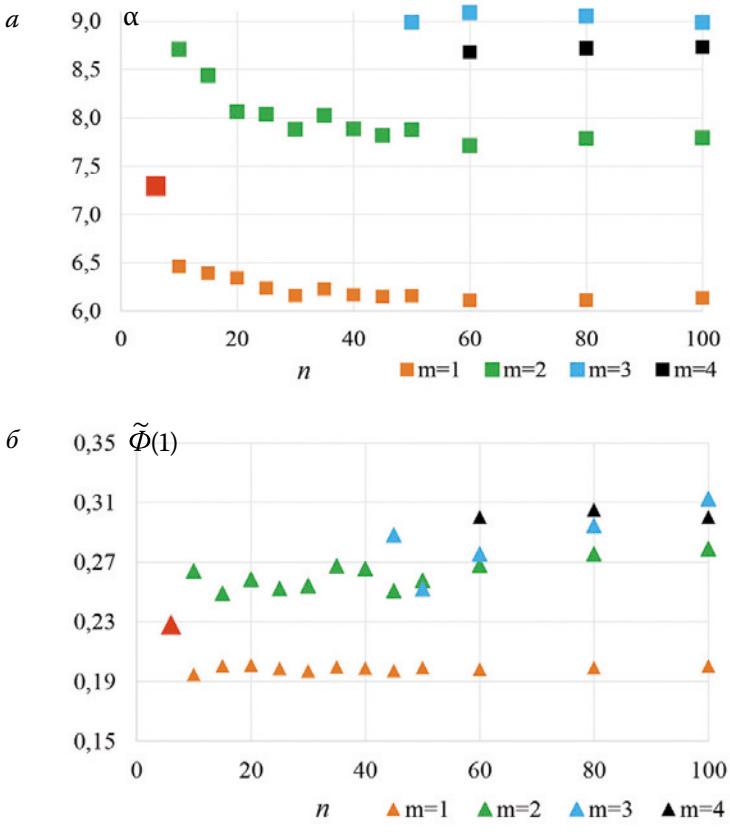


Fig. 6. Dependences of the parameters *a* – α; *b* – $\tilde{\Phi}(1)$ on the number of calculation points for various degrees of the polynomial for field measurements of the distribution of dissolved *Co* concentration.

The given dependences are in many respects similar to those considered earlier for the given test functions. Stable results of the Peclet number calculation (Fig. 6*a*) are observed for the second degree of the polynomial at $k \geq 20$, for the third degree of the polynomial at $k \geq 50$, for the fourth degree of the polynomial at $k \geq 60$. We can assume that in this case we see convergence of calculation values to some limiting values, which are determined by the mode of randomization of the initial data. Thus, if we assume one hundred measurements present under the given randomization conditions, we can speak about quite certain values of required parameters: $\alpha \approx 8.7$, $\widetilde{\Phi}(1) \approx 0.3$. Consequently, six measurements in this case are not enough and the obtained results of calculation ($\widetilde{\Phi}(1) \approx 0.22$, $\alpha \approx 7.3$) can be considered as the first approximation giving the estimated values of parameters.

RESULTS AND DISCUSSION

In mathematical models related to solving inverse problems for the advection-diffusion equation, the key issue is to reconstruct distribution of the specific mass flow rate of a pollutant on the site under study based on the data of its concentration distribution [1-9]. However, the problem of calculating the total mass flow rate characterizing the diffuse runoff is no less significant. This value is a well-defined calculation parameter for a given set of field measurements, along with the Peclet number, which is a priori unknown (this issue is discussed in detail in [10]). Thus, in this case, we should determine not the distributed, but the concentrated (effective) parameters. To some extent, it makes the problem more definite.

When solving inverse problems of the above class, there inevitably arises a question concerning the amount of necessary information about spatial distribution of dissolved pollutant concentration. Obviously, this is not a problem in mathematical modeling. However, when analyzing a set of field measurements it is crucial. Indeed, for the correct mathematical analysis when using regularization methods, the number of calculation points should be quite significant (hundreds of points). At the same time, a reasonable question arises: what number of points is necessary to obtain stable calculation results of the effective parameters indicated above?

The presented paper analyzes stability limits of results of calculating the total mass flow rate and the Peclet number with respect to the number of points used in numerical and field experiments. The number of design points and the polynomial degree for determining the total mass flow rate (5) were variable parameters. A set of independent test functions and randomization parameters discussed in [10] were used for calculation. Unknown parameters are found to be best determined with the third degree of polynomial, but for this purpose it is necessary to use about $k = 50$ points. Summarizing the obtained results, we can state the following:

– for each polynomial degree there is a bottom limit depending on the number of design points, below which calculation results cease to be stable;

– if the number of design points exceeding the specified limit, results of parameter calculation depend weakly on their number;

– the polynomial degree increasing, values of calculation parameters tend to some limit value.

The number of parameters is determined by the polynomial degree and the unknown Peclet number. As follows from calculations, the number of design points to obtain reliable and stable results should exceed approximately the number of unknown parameters by10 times (the number of coefficients plus the Peclet number), which corresponds to [21]. Indeed, for the third degree of the polynomial in all cases considered, the required number of points should be of the order of 40, for the second degree of the polynomial – of the order of 20.

When analyzing field measurements, there were obtained results similar to test calculations. Considering the results of six measurements to be the first approximation, calculation for the number of measurements was done. With the number of design points and the polynomial degree increasing, there was found a convergence of calculation values to some limiting values. Thus, we can assume that in this case, the criterion of correctness of calculation results is stability of values of definable parameters in relation to the number of design points and the polynomial degree.

CONCLUSION

In the presented work, for the first time we analyzed the influence of the number of design points on solving the inverse problem of determining the total mass flow rate of a dissolved pollutant and the Peclet number for pollution of small rivers. At randomization of calculation data, the Monte Carlo method was used for one thousand realizations of random deviation of pollutant concentration values.

In the course of numerical experiment for the given test functions it was found that with the degree of approximating polynomial increasing, there increases the required number of calculation points providing stability of required values. It was found out that in order to obtain reliable results, the number of design points should be approximately 10 times higher than the number of unknown parameters (in this case these are polynomial coefficients and the unknown Peclet number). The most reliable values for test functions under study correspond to the third degree of the polynomial, the number of design points being more than forty. The polynomial of the first degree gives a reasonable estimation of values of required parameters. Scenario calculations show that an increase in the number of design points causes convergence of definable values to some stable values.

The proposed method allows us to obtain stable calculation results of values determining diffuse runoff of conservative pollutants of small rivers at a small number of field measurements. The method can be used for monitoring the anthropogenic load on diffuse pollution of water bodies.

CONFLICT OF INTERESTS

The authors declare no relevant conflict of interests.

REFERENCES

1. Shen J., Jia J.J., Sisson G. Inverse estimation of nonpoint sources of fecal coliform for establishing allowable load for Wye River Maryland. *Water research*. 2006. Vol. 40 (18). 3333–3342. https://doi.org/10.1016/j.watres.2006.07.035.
2. Boano F., Revelli R., Ridolfi L. Source identification in river pollution problems: A geostatistical approach. *Water Resources Research*. 2005. Vol. 41 (7). 1–13. https://doi.org/10.1029/2004wr003754.
3. Permanoon E., Mazaheri M., Amiri S. An analytical solution for the advection-dispersion equation inversely in time for pollution source identification. 2022. *Physics and Chemistry of the Earth*. Vol. 128 (2). 103255. https://doi.org/10.1016/j.pce.2022.103255.
4. Rashedi K. A numerical solution of an inverse diffusion problem based on operational matrices of orthonormal polynomials. *Mathematical Methods in the Applied Sciences*. 2021. Vol. 44 (17). 12980–12997. https://doi.org/10.1002/mma.7601.
5. Sousa D.M., Roberty N.C. An inverse source problem for the stationary diffusion-advection decay equation. *Inverse Problems in Science and Engineering.* 2012. Vol. 20 (7). 891–915. https://doi.org/10.1080/17415977.2011.609466
6. Kulbay M., Mukanova B., Sebu C. Identification of separable sources for advection diffusion equations with variable diffusion coefficient from boundary measured data. *Inverse Problems in Science and Engineering.* 2017. Vol. 25 (2). 279–308. https://doi.org/10.1080/17415977.2016.1160396.
7. Jun K.S., Kang J.W., Lee K.S. Simultaneous estimation of model parameters and diffuse pollution sources for river water quality modeling. *Water Science and Technology*. 2007. Vol. 56 (1). 155–162. https://doi.org/10.2166/wst.2007.447.
8. Chaikovskii D., Zhang Y. Convergence analysis for forward and inverse problems in singularly perturbed time-dependent reaction-advection-diffusion equations. *Journal of Computational Physics.* 2002. Vol. 470. 111609. https://doi.org/10.1016/j.jcp.2022.111609.
9. Lukyanenko D.V., Shishlenin M.A., Volkov V.T. Solving of the coefficient inverse problems for a nonlinear singularly perturbed reaction-diffusion-advection equation with the final time data. *Communications in Nonlinear Science and Numerical Simulation*. 2018. Vol. 54. 233–247. https://doi.org/10.1016/j.cnsns.2017.06.002.
10. Filimonov V.Yu., Lovtskaya O.V. Semi-empirical method for calculating diffuse runoff of conservative pollutants in small rivers: Communication 1. Method for reconstructing the total mass flow rate. *Water Sector of Russia: Problems, Technologies, Management*. 2025. No. 1. P. 5–23. https://doi.org/10.35567/19994508-2025-1-5-23.

11. Filimonov V.Yu., Lovtskaya O.V., Zinoviev A.T. Calculation of dissolved pollutants mass flow according to the data of their concentration spatial distribution in the sites of small plaint rivers. *Eurasian journal of mathematical and computer applications*. 2023. Vol. 11 (4). 14–28. https://doi.org/10.32523/2306-6172-2023-11-4-14-28.
12. Filimonov V.Y., Zinoviev A.T., Lovtskaya O.V. Method for estimating the pollutant flow rate under diffuse pollution of small lowland rivers. *Mathematics and computers in simulation.* 2021. Vol. 182 (9). 807–818. https://doi.org/10.1016/j.matcom.2020.12.003.
13. Chueh Y.-Y., Fan C., Huang Y.-Z. Copper concentration simulation in a river by SWATWASP integration and its application to assessing the impacts of climate change and various remediation strategies. *Journal of Environmental Management*. 2021. Vol. 279. 111613. https://doi.org/10.1016/j.jenvman.2020.111613.
14. Liu Q., Cheng Y., Fan C. Pollution characteristics and health exposure risks of heavy metals in river water affected by human activities. *Sustainability*. 2023. Vol. 15 (10). 8389. https://doi.org/10.3390/su15108389.
15. Byrne P., Onnis P., Runkel R.L., Frau I., Lynch S.F.L., Edwards P. Critical shifts in trace metal transport and remediation performance under future low river flows. *Environmental Science & Technology*. 2020. Vol. 54 (24). 15742–15750. https://doi.org/10.1021/acs.est.0c04016
16. Ren J, Hao J, Tao L. Concentrations, spatial distribution, and pollution assessment of heavy metals in surficial sediments from upstream of Yellow River, China. *Environmental science and pollution research*. 2021. Vol. 28. 2904–2913. https://doi.org/10.1007/s11356-020-10349-3.
17. Hoang H.G., Lin C., Tran H., Chiang C.F., Bui X.T., Cheruiyot N.K., Shern C.C., Lee C.W. Heavy metal contamination trends in surface water and sediments of a river in a highly-industrialized region. *Environmental Technology & Innovation*. 2020. Vol. 20. 101043. https://doi.org/10.1016/j.eti.2020.101043.
18. Lu Q., Bian Z., Tsuchiya N. Assessment of heavy metal pollution and ecological risk in river water and sediments in a historically metal mined watershed, Northeast Japan. *Environmental monitoring and assessment.* 2021. Vol. 193 (12). 814. https://doi.org/10.1007/s10661-021-09601-1.
19. Markich S.J., Brown P.L. Relative importance of natural and anthropogenic influences on the fresh surface water chemistry of the Hawkesbury Nepean River, south-eastern Australia. *Science of the total environment*. 1998. Vol 217 (3). 201–230. https://doi.org/10.1016/s0048-9697(98)00188-0.
20. Filimonov V.Y., Lovtskaya O.V., Zinoviev A.T. Diffuse pollution on small rivers. Features of distribution of dissolved pollutant concentrations. 2021. *Eurasian journal of mathematical and computer applications*. 2021. Vol. 9 (3). 39–52. https://doi.org/10.32523/2306-6172-2021-9-3-39-52.
21. Draper N.R., Smith H. Applied Regression Analysis: Wiley, New York, 1998. 736 p.

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